Summary and conclusions

In this thesis, the theories for wavefield-modeling and inverse problems in seismic tomography are improved. Seismic tomographic experiments are based mostly on ray theory which is valid for the description of wave propagation in media where velocity anomalies are larger in size than the wavelength and the Fresnel zone (see Fig. 1.1 in the introduction of the thesis, chapter one, for an explanation of the physical parameters). Nonetheless, the newest surface wave models of the Earth have characteristic lengths of inhomogeneity of approximately the same order as the wavelength and the Fresnel zone. This observation poses a problem from a methodological point of view, as it is inappropriate to use an approximative theory (namely ray theory) that is only valid under conditions that are not satisfied in the final result (e.g. the tomographic model). Instead, it is necessary to take the scattering of waves into account in the theory for wave propagation so that inhomogeneities with length-scales comparable with or smaller than the Fresnel zone are modeled correctly. Second, a common problem in the inversion of seismic data is that heterogeneities smaller than a certain (arbitrary) length scale are neglected due to practical reasons. However, the truncation of the allowed length-scale in surface wave tomographic experiments may introduce a systematic error in the surface wave models of the Earth.

It is shown that the timeshift can be expressed as a volume integration of the slowness perturbation field multiplied by the Fréchet kernel due to the scattering of waves. The Fréchet kernel is an analytical function in wave experiments where the straight ray approach holds. The developed scattering theory is therefore just as easy to apply as the ray theoretical approach using the great circle approximation in surface wave tomographic experiments. The Fréchet kernel depends on the experimental parameters, such as the distance between the source and receiver, the reference slowness field and the frequency content of the measured wavefield. It is shown in chapters three through five that the Fréchet kernel for finite-frequency waves has the maximum sensitivity to slowness perturbations off-path from the geometrical ray. For waves propagating in three dimensions, the sensitivity to slowness perturbations vanishes on the ray path (see Fig. 4.1B in chapter four). This is a counter-intuitive result compared with ray theory which predicts non-zero sensitivity to slowness perturbations only on the geometrical ray.

The formation of caustics is significant in small-scale structured media with strong slowness perturbations (chapter two). However, multiple arrivals associated with caustics arrive after the ballistic wavefield due to causality. It is appropriate to use the theory for the scattering of waves even though triplications form in the propagating wavefield.

The developed scattering theory is a generalisation of ray theory. One obtains the same result using either ray theory or diffraction theory in the parameter regime where ray theory is valid. In contrast, only the theory for finite-frequency waves predicts the timeshifts retrieved from a finite-difference 2-D modeling wave experiment (chapter three and four) and from a physical 3-D experiment with ultrasonic waves propagating in samples of granite (chapter four) for which scattering effects are important.

The theory for the scattering of waves is applied in global surface wave tomography for Love waves between 40 s and 150 s (chapter five). In the global surface wave experiment, it is found through a synthetic experiment that present-day global high-resolution surface wave tomographic models are at the limits of the application of the ray theoretical great circle approximation. The great circle approximation is valid in surface wave tomography as long as the characteristic length of slowness anomalies is larger than 1300 km and 2000 km for Love waves with the periods of 40 s and 150 s, respectively. Phaseshift measurements for Love waves between 40 s and 150 s are applied in separate inversions using ray theory and scattering theory. The estimated tomographic surface wave models derived from ray theory and scattering theory are similar, because a restrictive regularisation condition is incorporated in the inversion so that structures with length-scales smaller than the Fresnel zone are mostly suppressed. However, it is important in future global surface wave tomographic experiments to apply surface wave scattering theory instead of ray theory in order to obtain higher resolution models of the Earth than is presently possible.

Surface wave scattering theory is applied together with spectral leakage theory in an inversion of phase velocity measurements for Love waves between 40 s and 150 s (chapter six). The phase velocity models from the spectral leakage inversion are obtained without any damping in use, and they are therefore compared most correctly to the surface wave models from a common least squares solution without applying any regularisation condition. It is found that the estimated surface wave models for Love waves between 40 s and 150 s from the undamped spectral leakage solution correlate better with tectonic features such as plate boundaries, ridges and trenches than the phase velocity models retrieved from the common least squares solution without using any regularisation condition.

The conclusions above do not contradict the application of ray theory in tomographic wave experiments, but this thesis demonstrates examples that ray theory is not always appropriate for the description of propagating waves in media with small-scale heterogeneity. When imaging techniques are based on ray theory in a given tomographic wave experiment, it should be verified in the inverted model that the conditions for ray theory are satisfied. For complex media where the conditions for ray theory are not valid, the modeling of wave propagation should be based on scattering theory instead of ray theory.

There are several natural extensions of this thesis research. First, the scattering theoretical approach in chapter three is made as general as possible, thereby making it feasible to incorporate the non-ray geometrical effect in global body wave tomography, in regional surface wave tomography, in seismic exploration (e.g. crosswell tomography, vertical seismic profiling, reflection seismic experiments and migration theory), in medical imaging and in ocean acoustics. Second, the scattering theory in the thesis is limited to the case of isotropic, homogeneous reference media. It would be of interest to generalise the developed diffraction theory to include heterogeneity and anisotropy in the background medium as well. Third, the Rytov approximation is here applied on the acoustic wave equation. In the case of surface wave scattering theory in chapter five, it is appropriate to use the Rytov approximation on surface waves because the approach is limited to transmitted and unconverted surface waves. It should be investigated how to generalise the Rytov approximation to elastic wavefields, so that mode conversion between surface waves or P-to-S wave conversion and vice versa can be taken into account in future tomographic wave experiments. Fourth, it is observed in synthetic aperture radar (SAR) experiments that it is possible to obtain high-resolution images of the Earth's surface with length-scales of heterogeneity that are much smaller than the width of the Fresnel zone of radar waves. The theory of the scattering of waves presented in this thesis is related to the concept of Fresnel zones, because the finite-frequency of waves is taken into account. It would be interesting to construct synthetic experiments which are identical to the SAR-experiment or to high-resolution seismic exploration experiments so that the limits of resolution in tomographic wave experiments can be investigated analytically. Lastly, the work about scattering theory by Dahlen *et al*. (2000) and Hung *et al*. (2000) indicates that non-linear effects are important (see the introduction in chapter one for references). In contrast, the scattering theory based on the Rytov approximation as found in this thesis is linearised. It is important to investigate the role of non-linearity on the propagation of waves in media where scattering effects are important.