CHAPTER 1

Introduction

Over the last decades, the study of climate variability has attracted ample attention. Temperature records show signs of global warming, starting around the end of the nineteenth century. The observation of this structural climatic change has led to questions about its causes and the mechanisms involved. Perhaps the most important issue is, to what extent climatic change is due to anthropogenic influence, connected to the industrial revolution, and to what extent to natural variability. This issue is becoming more and more important, as speculations arise about the link between climatic change and catastrophes such as floods and droughts. The task to understand interactions in the complex climate system is particularly difficult because of the lack of observational data, spanning a period of time typical for natural climate variability.

One way around this problem is to represent the earth's climate in a computer model, as a set of prognostic equations. By means of numerical integration the past and future climate can then be reconstructed. Also, numerical experiments can be conducted in which a number of quantities are kept fixed in order to investigate the sensitivity of the climate system to isolated physical effects.

A disadvantage of this approach is that, if the model under consideration is to faithfully represent the real climate system, it has to be large in terms of the number of degrees of freedom. Depending on their extent of realism, so called General Circulation Models (GCM's) have from a few thousand up to a few million degrees of freedom. This puts them out of reach of the ordinary analysis of dynamical systems theory. Instead, statistical analysis is used to study the model's output. The measurement of, for example, correlation coefficients, combined with the physical theory behind the processes represented in the model may lead to insight into the mechanism of climatic change and variability. The mathematical structure behind it, however, remains unclear. As a rule of thumb, the bigger the model, the harder it is to investigate its dynamics mathematically. Such a mathematical investigation can validate conclusions drawn on basis of experiments conducted with the model, as it focusses on stability of its behaviour to perturbations and genericity of its behaviour in a wider class of models.

The problem, that mathematical analysis is harder for more realistic models, also frustrates the study of weather, restricted to a time scale of days or weeks. As weather has a considerable impact on daily life, meteorology dates back to long before the computer era. Numerous approximations and simplifications have been devised to render the equations for atmospheric flow solvable by hand. In classical studies such as Philips [1954]; Charney [1959]; Lorenz [1960] solutions are obtained by imposing symmetries, considering limits of physical parameters and exploiting perturbation theory. Their approach and results are now common knowledge in meteorology.

Computers made it possible to investigate solutions, out of reach of perturbation theory. At first, however, numerical computations were slow and painstaking. Forward integration of a set of three prognostic equations was a considerable task. Edward Lorenz had such a model at hand. He split the forward integration into parts and had them overlap in order to check the numerics. According to the popular anecdote this is how he came across the phenomenon of sensitive dependence on initial conditions, nowadays a major paradigm in meteorology and climatology. On a present day desktop computer, the whole computation he had in mind would take less than one second and this sensitive dependence may well go unnoticed.

Thus, the study of the equations governing atmospheric flow, without resort to massive numerical simulations, has led to many fundamental insights in meteorology. The study of extremely simplified climate models, as presented in this thesis, should likewise lead to an understanding of the mechanisms of climatic change. One feature climate models share is the presence of widely different time scales. Components of the climate system, which can often be considered fixed in the context of weather forecasts, have to be taken into account explicitly. Throughout this thesis the emphasis will be on the question to what extent the slow time scales play a role in the model's dynamics. In climate models, the slow time scales may be related to, e.g., ice sheet dynamics, variations in solar heat flux or ocean dynamics. In this work, the slow time scales will only be related to ocean dynamics and the fast time scale to atmospheric dynamics. The question is thus if the ocean plays an active or a passive role in the combined system [Marotzke, 1994].

An important class of extremely simplified models (for a review, see Olbers [2001]) is formed by Low-Order Models (LOM's). Low-order refers to the number of degrees of freedom. In conformance with mathematical literature I will refer to models of low order as those with a number of degrees of freedom not much greater than 10. This is a subjective definition which serves its purpose in the context of this work. The equations governing the dynamics of the atmosphere and the ocean are partial differential equations, with an infinite number of degrees of freedom. I will loosely refer to them as Fluid Dynamical (FD) equations, a notion which will be specified in chapters 2 and 4.

The set of LOM's can be subdivided in *conceptual* and *scalable* models. Conceptual models are formulated in an ad hoc fashion. The terms in the prognostic equations are chosen such that they represent certain isolated physical processes. Examples of such models are Stommel's model for the ocean [Stommel, 1961], in which the ocean is thought of as a number of boxes, connected by pipes, and the Daisyworld model [Watson and Lovelock, 1983], in which the face of the earth is covered by black and white daisies, competing for space. Scalable models are derived from a FD model by means of Galerkin truncation. The variables of the FD model, such as temperature and velocity fields, are projected onto a finite number of basis functions, which describe spatial structures. The expansion coefficients are the variables of the LOM. Some basis functions, common in fluid dynamics, are:

- Fourier modes, as in Lorenz [1963], a description of the three degrees of freedom model mentioned above, and in chapter 2 of this thesis,
- orthogonal polynomials, as in Maas [1994], see chapter 4 of this thesis, and
- eigenfunctions of covariance operators, here referred to as Empirical Orthogonal Functions (EOF's), see Preisendorfer [1988].

These sets of basis functions can be ordered by the spatial scale they take into account. In the case of Fourier functions, for instance, this ordering is simply given by the wave number. The spatial scales are tied to time scales through the typical velocity of the fluids or masses of air under consideration. Hence, the reduction to finite order is, at the same time, a selection of relevant time and spatial scales. Contrary to conceptual models, the dynamics of scalable models is determined by the FD model. Also, the number of degrees of freedom of scalable models can be increased at will in order to investigate the robustness of their behaviour. Thus, stronger statements about the real climate system can be inferred from the study of scalable models.

There is no definite answer to the question, to what extent the solutions of a LOM represent solutions of the parent FD model. In general, experience shows that the solutions of a FD model settle down on a finite dimensional, attracting set in phase space. For certain FD models, rigorous analysis yields an upper bound for the dimension of this set [Temam, 1988]. This means, that the dynamics of the FD model can at least be captured by a model of finite order. Such upper bounds, however, are large compared to the definition of low-order put forward here. Therefore, we can not expect the LOM's to quantitatively reproduce solutions of the FD model.

The question, how to represent the effect of omitted degrees of freedom in a LOM is known as the closure problem. Some of the proposed solutions are inclusion of stochastic forcing terms in the LOM [De Swart and Grasman, 1987], statistical optimisation of the LOM's coefficients [Achatz and Branstator, 1999] and approximation of inertial manifolds by means of a modified Galerkin method [Foias et al., 1988]. In this thesis, I will not address the closure problem. The LOM's studied here are not intended to yield quantitatively correct predictions about the real climate system. The Galerkin method is regarded as a means to select time and spatial scales, and thereby the physical processes of interest. It is checked, however, that the LOM's output has the right order of magnitude.

As mentioned above, the low-order climate models in this thesis are coupled ocean-atmosphere models. The atmosphere model, studied here, was introduced by Lorenz [1984]. Lorenz only hinted at the possibility to derive the model as a Galerkin truncation of a FD model. In Chapter 2 of this thesis this link is made explicit. In doing so, the physics behind the model and its scaling are described in detail. It is shown, that the Lorenz-84 model describes the jet stream in the mid-latitude atmosphere, and planetary waves, which can grow if the jet stream becomes dynamically unstable [Peixoto and Oort, 1992, chapter 7]. The typical time scale, associated with variability of the jet stream, also called the synoptic scale, is about one week. In subsequent chapters this will be the fast, atmospheric time scale.

The Lorenz-84 model will be coupled to two different low-order ocean models. In chapter 3, it is coupled to Stommel's two box model [Stommel, 1961]. Stommel's model mimics the large scale overturning, or thermohaline circulation in the North Atlantic ocean [Peixoto and Oort, 1992, chapter 8]. The typical time scale of variability of the thermohaline circulation is of the order of centuries. This will be the longest time scale in the coupled models.

In chapter 4, the Lorenz-84 model is coupled to an ocean model formulated by Maas [1994]. Contrary to the two box model, this is a scalable model. Consequently, considerable effort is put into a physical description of the coupling. Apart from the overturning circulation, Maas' model is capable of representing a wind driven gyre. The coupling works in two ways: through exchange of heat at the surface and through wind shear forcing. The latter acts on a time scale in between the fast atmospheric time scale and the slow overturning time scale. The intermediary time scale is set to about one year.

The LOM's in this thesis are sets of coupled, nonlinear, Ordinary Differential Equations (ODE's) on one, two and three widely separate time scales, respectively. These can be analysed with the aid of dynamical systems theory [Wiggins, 1990]. The emphasis will be on bifurcation analysis, i.e. the analysis of the dependence of the qualitative behaviour of the models on their parameters [Kuznetsov, 1998]. Also, the time scale separation leads to the presence of small parameters in the equations. The consequences for the behaviour of the coupled models are briefly explored by means of singular perturbation theory [Wiggins, 1994].

In chapter 2, the bifurcation structure of the Lorenz-84 model is investigated in some detail. Here, the focus is on the routes to chaotic behaviour in this model. In the subsequent chapters, the bifurcation structure of the coupled models is then compared to that of the uncoupled models. Keeping in mind the issue of time scale interaction, two differences stand out. In both coupled models, prominent intermittent behaviour is observed. This behaviour occurs near a point in parameter space at which the stability of periodic motion is lost. The slow subsystem, i.e. the ocean model, repeatedly pushes the fast subsystem, i.e. the atmosphere model, through a sequence of bifurcations. Thus, the ocean model plays an active role in the coupled system. Secondly, in the Lorenz-Maas model a periodic solution is shown to exist, with a period on the slow, overturning time scale. The atmospheric variables are at instantaneous equilibrium with the feedback of the ocean, and the behaviour of the coupled model is thus dictated by internal ocean dynamics. Both these phenomena occur near a critical point of the coupled system, in agreement with the general idea that in climate models the slow components can play an active role near such critical points and are passive otherwise.

There is not much literature on low-order models of ocean-atmosphere interaction. Models, studied in this field, are often extensions of the box-type approach [Huang and Stommel, 1992; Nakamura et al., 1994; Rivin and Tziperman, 1997; Titz et al., 2002]. To my knowledge, the Lorenz-Maas model, introduced in chapter 4, is the only scalable low-order model which includes overturning and wind driven circulation in the ocean. Accordingly, a number of extensions of this model is proposed, such as the inclusion of salinity and wave-wave interaction in the atmosphere.

The chapters of this thesis are based on the following papers:

- Chapter 2:
  Veen, L. van [2002] "Baroclinic flow and the Lorenz-84 model", to appear in the *Internat. J. Bifur. Chaos.*
- Chapter 3:
  Veen, L. van, Opsteegh, T., and Verhulst, F. [2001] "Active and passive ocean regimes in a low-order climate model", *Tellus* **53A**, 616-628
- Chapter 4:
  Veen, L. van [2002] "Overturning and wind driven circulation in a low-order ocean-atmosphere model", submitted to *Dyn. Atmos. Oceans.*

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