Chapter 1

Introduction and outline

1.1 Context

Seismic waves are widely used to study the earth's interior by means of measurements on or close to its surface. Global seismology is concerned with features along the entire depth range of the earth and therefore relies almost exclusively on the high energy waves generated by natural earthquakes. For shallow depths it is feasible to use artificial sources, such as explosives or vibrators, and to set up controlled seismic experiments. Depending on the types of sources and recording equipment, seismic experiments can provide information on subsurface properties at depths ranging from a few metres for engineering purposes, down to the deep crust and upper mantle.

Reflection seismics

The work described in this thesis is relevant for seismic exploration for oil and gas. Hydrocarbon reservoirs are typically found in sedimentary basins, at depths of the order of a few kilometres. The most commonly applied and successful experiment in this geological setting is the *seismic reflection experiment*, in which both sources and receivers are spread out on the surface. The success of the reflection experiment is primarily due to the typically layered structures of sedimentary basins, which reflect (scatter) seismic waves generated at the source back to the surface, where they are recorded by the receivers.

Despite the relative placidity of the environments in which sedimentation usually takes place, sedimentary basins can exhibit very complex structures. These are usually caused by later tectonic events such as folding, faulting and intrusion of salt or basalt. Complex structures are often the most interesting part of such a subsurface because these may contain structural traps that give rise to accumulation of hydrocarbons. The processing and interpretation of seismic data in complex structures are difficult and remain a challenge.

The seismic inverse problem

The general goal of any seismic method is to obtain quantitative information on certain subsurface properties. In reflection seismics, the desired information has for a long time been mostly geometric, in the form of a structural image. Nowadays, however, a certain amount of information regarding physical properties is usually required as well. This information may, often in combination with geological data obtained in boreholes, provide clues regarding rock types and relevant parameters such as porosity, permeability and fluid content.

In mathematical terminology the problem of transforming the seismic measurements, *the data*, into a set of quantitative subsurface properties – *the model* (parameters) – is called an *inverse problem*. As indicated by its name, the inverse problem has a counterpart: the *forward problem*. This problem concerns the expression of data in terms of the model parameters by means of a mathematical model for the physics involved in the experiment. The methodology used for solving the inverse problem relies strongly on the formulation of the forward problem.

For most practical situations in seismics the earth may be considered to be an elastic medium. The elastic wave equation should therefore be an adequate mathematical model for the forward problem. In general, however, the relationship between a solution of the wave equation (the measured data) and its coefficients (the model parameters) is strongly non-linear. This non-linearity makes the inverse problem very hard to be tackled directly.

A common approach to addressing such non-linear inverse problems is by means of perturbations. The model parameters sought are then defined to be small perturbations to a known (reference) model. The forward problem can then be linearised, thereby facilitating the corresponding inverse problem. In reflection seismics a practical approach is to define a smooth reference model and consider sharp features, such as interfaces and faults, as the perturbations to be inverted for. Some common terms that are used for this kind of inversion are *migration*, *imaging*, and *inverse scattering*. The latter name relates to the interpretation that the sharp features act as scatterers of the wave field propagating in the smooth reference model.

The most challenging aspect of the inversion of seismic reflection data is, probably, to determine the smooth reference model for which the linearised inverse scattering procedure may give a reliable result. This procedure is also known as *velocity analysis* and is typically done by means of *tomography*. Whereas the inverse scattering is usually a one step procedure, the tomographic inversion requires iteration. The quality of updates in the reference model is usually estimated by means of a measure of the coherency (focusing) of the result of the inverse scattering.

Both inverse scattering and tomographic inversion rely on many forward calculations of wave propagation in the background medium. These forward calculations can be performed in a number of ways, for example by finite difference or spectral methods. Some of the most popular forward modelling methods are based on *ray theory.* This theory uses a high frequency approximation of the wave equation and therefore has a limited validity. Nevertheless, its limitations are relatively well understood, and so are those of the inverse methods based on it. Moreover, ray methods are generally very efficient compared to the alternative modelling methods.

References

A number of general references for the context of this thesis are recommended: Lee et al. (2002) and Aki and Richards (2002) on global seismology, Sheriff and Geldart (1995) on reflection seismics, Brouwer and Helbig (1998) on engineering seismics, Duff (1993) on physical geology, Tarantola (1987) on inverse problems, Bleistein et al. (2001) on imaging, and Červený (2001) on seismic ray theory.

1.2 Motivation

One of the characteristic features of ray methods is that the calculations are not performed directly in terms of the spatial coordinates of the medium. Although the seismic ray equations are derived from the elastic wave equation they do not share the same computational domain. The wave equation is a partial differential equation for displacement as a function of the spatial coordinates. The kinematic ray equations, on the other hand, are equations for the ray position as a function of a single parameter such as time or arc length, referred to as the *flow parameter.* The ray described by these equations is a flow line of wave energy in the high frequency approximation.

To study an entire wave field it is necessary to consider an ensemble of rays, or *ray field*, parameterised by the *ensemble parameters.* In the case of a point source, for example, these parameterise the initial ray directions at the source location. Together, the flow parameter and the ensemble parameters form an internal coordinate system for the ray field and are referred to as *ray field coordinates.*

The ray-theoretical wave field is parameterised by travel time and amplitude functions that are determined – along with the spatial coordinates – as a function of the ray field coordinates. To evaluate the travel time and amplitude at a given spatial location one needs to know its corresponding ray field coordinates. In other words, one needs to evaluate the mapping from spatial to ray field coordinates: the *ray field map.*

If the medium is sufficiently complex for the ray field to develop caustics and multi-pathing of rays, the ray field map becomes multi-valued. This multi-valuedness is a source of many practical problems in the application of ray methods to both forward and inverse wave propagation problems. In the forward calculations, for example, it is difficult to get an unambiguous and accurate estimate of all arrivals. This is especially the case in the neighbourhood of caustics, where the number of arrivals changes abruptly. In inverse methods multi-valued ray field maps are cumbersome to work with. The interpolation of multi-valued maps, for example, is a notorious problem.

In this thesis a number of new approaches to the calculation of ray fields and ray field maps are presented. The central theme is the solution of the practical problems encountered in smooth but *complex media*, i.e., media that give rise to wave front folding and associated multi-pathing. The ultimate aim of the presented material is to enhance the efficiency of seismic inverse methods, by enhancing the efficiency of the forward calculations. Therefore, particular emphasis is placed on the applicability of the results to seismic inverse methods.

1.3 Outline

The essential background in seismic ray theory is provided in **Chapter 2**. The emphasis is on explanation and interpretation of the basic ingredients, rather than on derivation of equations, which are well covered in the existing literature. The concepts of ray fields and ray field maps are introduced as well.

In **Chapter 3** a novel approach to the calculation and representation of ray field maps is introduced that is particularly useful in cases where ray field maps are needed for a dense distribution of sources at an acquisition surface. This is the case in most seismic experiments such as reflection seismics and borehole tomography. For such source distributions it is suggested to construct a single ray field map in an extended space of spatial coordinates and angles, rather than a number of maps in the spatial domain for a range of acquisition coordinates.

A ray field map in the position/angle domain is single-valued, regardless of the complexity of the medium. The ray field information is organised by angles at depth rather than by points of emergence at the surface, which makes the maps particularly suitable for use in modern seismic imaging methods. It is shown that to calculate these maps it is not necessary to trace rays up towards the acquisition surface, which would involve an unacceptable increase in the computational burden. Instead, existing algorithms that trace downwards can be adapted to work in the position/angle domain.

Interpolation is an important tool in both the construction and the application of ray field maps. A new technique for accurate interpolation using derivative information is presented in **Chapter 4**. It is a hybrid of extrapolation to arbitrary order and linear interpolation, and combines the advantages of both methods. Through a modification of the coefficients of the Taylor expansion, extrapolations from a number of locations can be combined to obtain a polynomial order of accuracy that is one higher than that of a single conventional Taylor expansion.

The formulation of the method is very general, and it can be used both with regular and irregular data distributions in arbitrary dimensional spaces. In regular grids it is possible to use finite difference estimates of derivatives if these are not available independently. The interpolation technique is expected to be useful in many applications and is used at various locations in this thesis.

In **Chapter 5** a ray field construction and mapping algorithm is developed that extends and refines existing wave front construction methods. A modular setup and a hierarchical description of the geometrical structure of the ray field make the algorithm widely applicable. It can be used for the calculation of ray fields and ray field maps in smooth 2-D and 3-D, isotropic and general elastic media.

For ray field mapping in the spatial domain two refinements are proposed that enhance the accuracy and the completeness of the maps by higher order interpolation and improved delineation of caustics. Both refinements are easily included in existing wave front construction methods to enhance the efficiency.

Motivated by the success of wave front construction methods in the spatial domain, the applicability in the position/angle domain is investigated as well. The unfortunate conclusion is that ray field construction in its current form is not suitable for that domain, due to the type of deformation in the geometrical structure of the ray field. In the position/angle domain this deformation is primarily shear-like whereas the algorithm is designed for spreading ray fields in the spatial domain.

A better algorithm for the calculation of ray field maps in the position/angle domain is developed in **Chapter 6**. It is based on the observation that in the position/angle domain the ray field maps are single-valued and that the geometrical spreading is very limited. This implies that the two most important reasons for developing wave front construction methods in the spatial domain are absent in the position/angle domain. Instead, it is possible to use the more primitive – but more efficient – paraxial ray methods.

The one-to-one mapping between position/angle coordinates and ray field coordinates can be exploited in practical applications. Calculations that are typically performed in terms of ray field coordinates can now be performed in terms of position/angle coordinates and the other way around. **Appendix A** shows that this may be advantageous in tomography. If the ray field map is known for a reference model, the cost function gradient can easily be calculated for an arbitrary parameterisation of the model perturbation, using the new concept of a cost function sensitivity kernel.

The change of coordinates from ray field to position/angle coordinates can be exploited even further. In **Appendix B** it is shown how the theory of Chapter 3 may be used to derive equations for the evolution of the ray field coordinates in terms of the position/angle coordinates. These equations may be used as the basis for a finite difference algorithm that calculates the full ray field information for a range of sources directly on a grid in the position/angle domain. This procedure avoids both the explicit mapping step that is usually required after ray tracing and the interpolation of medium properties at arbitrary spatial locations.

Finally, **Appendix C** presents an algorithm for the calculation of ray fields in smooth 2-D media, using a pseudo-spectral expansion of the wave front. This line of research was abandoned in favour of the ray field map methods described above. Nevertheless, it is presented here because its development provided useful insights for the ray field map approach (e.g., Appendix B) and some of its features may be useful in other applications.