Chapter 7

General discussion and concluding remarks

The research described in this thesis covers various aspects of the forward calculation of ray fields and ray field maps. The central theme is the solution of problems encountered in smooth but complex media, i.e., media that give rise to wave front folding and associated multi-pathing of rays. The ultimate aim of the presented material is to enhance the efficiency of seismic inverse methods, by enhancing the efficiency of the forward calculations. Particular emphasis has been placed on the applicability of the ray tracing results to seismic inverse methods.

The results can be broadly categorised into four subjects, which will be discussed in the following sections.

7.1 Pseudo-spectral ray tracing

One of the first objectives of this research was to develop an efficient algorithm for the calculation of ray fields in smooth media, and to make this part of a perturbation method that would be able to handle variation in the number of ray arrivals under perturbation of the medium. The algorithm has two distinctive features; first, the wave fronts are approximated by a pseudo-spectral expansion, and second, the parameterisation of the wave fronts is adjusted dynamically to maintain a homogeneous sampling of the ray field throughout the medium.

Although the approach is conceptually attractive, obtaining a stable and efficient algorithm proved to be difficult. The method appears to be more complex and less flexible than, for example, the ray field construction method presented in Chapter 5. Furthermore it cannot easily be extended to 3-D.

Since it is unlikely that pseudo-spectral ray tracing will be able to compete with ray field construction (for speed) it was decided to abandon this line of research in favour of the more practical ray field map methods. However, we have included the work on the pseudo-spectral ray tracer in this thesis, as Appendix C, because its development provided useful insights for the ray field map approach – e.g., compare the dynamic parameterisation mechanism in Appendix C with the method proposed in Appendix B – and some of its features may be useful in other applications.

7.2 Intrapolation and the Dutch Taylor expansion

Interpolation methods play an important role in various stages of the calculation and application of ray fields and ray field maps. In many cases, not only the values of the function to be interpolated are known at the data points, but also the first or even second order derivatives. This is the case, for example, for the interpolation of travel times in ray field cells, where usually the slowness vector – the gradient of travel time – is available. If paraxial ray tracing is performed, even the second order derivative of travel time – related to the wave front curvature – may be evaluated. Also, for the interpolation of ray field maps on regular grids the derivatives can easily be determined using finite differences.

Many interpolation methods have been developed and applied in many fields of science. However, an extensive search through the literature and the world wide web did not yield a simple and generally applicable method to incorporate derivative information in interpolation to enhance the accuracy. It is possible to construct interpolating polynomials that fit derivatives as well as the function values in arbitrary data point configurations, but this is usually impractical. The analysis required to obtain the interpolants is time consuming and, thus, computationally expensive, and the interpolants are necessarily of higher order, which again means that they are expensive to evaluate, and the risk of enhancing errors in the data is high.

The lack of a suitable method for accurate interpolation using derivative data led to the development of the *intrapolation* technique described in Chapter 4. Intrapolation is a hybrid of extrapolation to arbitrary order and linear interpolation and combines the advantages of both. The extrapolation is done by a modification of the Taylor expansion, for which we coined the term *Dutch Taylor expansion*. The order of accuracy of intrapolation is one higher than that of a single conventional Taylor extrapolation that uses the same amount of derivative information.

Since both extrapolation and linear interpolation are easily performed in arbitrary dimensional spaces and on arbitrary data distributions, intrapolation should be applicable to a wide variety of problems. In the context of ray methods it is immediately applicable to the interpolation of ray field maps on rectangular grids (see Section 4.4.2). It has also proven its value in the forward calculation of travel time maps in the spatial domain: first, as a method for interpolating travel times within ray field cells with second order accuracy, and second, for interpolating additional ray field points to improve the delineation of caustics (see Section 5.4).

The applicability of the Dutch Taylor expansion in a wider context was investigated in Section 6.2.3. In theory, the expansion can also be used to enhance the accuracy of regularisation by means of averaging integrals. In practice, however, the averaging integrals are evaluated with insufficient accuracy for the Dutch Taylor expansion to have the desired effect.

7.3 Ray field construction in the spatial domain

The calculation of ray-theoretical Green functions on spatial grids is a challenging computational task, especially in complex media, where both strong geometrical spreading and multi-pathing are complicating factors. A very efficient and popular class of algorithm for this type of calculation is called wave front construction. In Chapter 5 a number of extensions and refinements to wave front construction have been presented under the name *ray field construction*.

The proposed algorithm generalises the wave front construction methods by allowing an arbitrary number of ensemble parameters, i.e., degrees of freedom in the initial conditions of the rays in the ray field. This has been accomplished by defining a hierarchical description of the ray field structure and a recursive approach to the ray field propagation. As a result, this algorithm can not only be used for the usual ray tracing in 2-D or 3-D media but also for ray field construction in the position/angle domain, which is discussed in Section 7.4.

A modular setup makes the algorithm highly adaptable to various types of model parameterisation. Both isotropic and general elastic (anisotropic) smooth models are currently supported. Some examples have been shown for media with vertical transverse isotropy (VTI).

For applications in the spatial domain we propose two refinements of the existing ray field mapping methods (Section 5.4). The use of intrapolation, see above, increases the accuracy of travel time mapping within the ray field cells to second order. Also, a refinement for the ray field sampling in the neighbourhood of caustics is proposed under the name *accurate caustic delineation*. Both refinements allow a smaller number of rays in the ray field, which enhances the efficiency of the ray field construction.

7.4 Ray field maps in the position/angle domain

7.4.1 Theory

A large part of this thesis is devoted to the development of theory and algorithms for the construction of ray field maps in the position/angle domain. This new approach for the representation and calculation of ray field information, introduced in Chapter 3, is particularly useful if ray fields have to be calculated for a dense distribution of sources and/or receivers at an acquisition surface. This is a common situation in seismic imaging experiments such as reflection seismics and borehole tomography.

A single ray field map in the position/angle domain contains the ray field information associated with a range of acquisition points. As such, it provides an alternative to a large number of maps in the spatial domain, each containing the ray field information associated with a single acquisition point. The advantages of this approach are particularly pronounced in complex media, where ray field maps in the spatial domain become multi-valued and cumbersome to work with in practical applications. As shown in Chapter 3, a ray field map in the position/angle domain is single-valued, regardless of the complexity of the medium.

Moreover, modern approaches to ray-based imaging in complex media deal with the problems of multi-pathing by parameterising the imaging integrals in terms of scattering angles and azimuths at depth. This requires the ray field information to be organised by angles at depth, exactly as it is provided by a ray field map in the position/angle domain. However, in contrast to what is commonly assumed, obtaining this information does not require the tracing of rays from the image points up towards the acquisition surface. Instead, existing algorithms that trace downwards can be be adapted to work in the position/angle domain, leading to a considerable gain in efficiency. For further discussion on algorithms see Section 7.4.2.

Whether or not ray field maps in the position/angle domain will be able to play an important role in tomorrow's imaging processes not only depends on the availability of practical algorithms to calculate them, but also on the future status of angle domain imaging methods. Although the theoretical advantages of these methods are generally recognised and preliminary case studies show encouraging results, practical application, especially in 3-D, still faces some challenges.

The greatest practical problem for angle domain imaging currently seems to be related to the data flow. The evaluation of imaging integrals in terms of angles at depth requires more or less random data access. Since the total volume of data in 3-D experiments is extremely large this may result in a dramatic increase in disk I/O. Solutions to this problem may perhaps be found in alternative ways of storing the data, another possible future subject of research.

Ray-based imaging methods also face the challenge from competing "wave-equation imaging" methods. Traditionally, these methods, which are based on more accurate wave theories, have not been used as often for production work in 3-D because of their high computational cost. However, continuing advances in both algorithms and computing power have pushed the wave equation methods (almost?) within the range of feasibility. Despite this competition, ray-based imaging techniques will probably remain of interest for some time to come as the preferred tools for target-oriented approaches and migration velocity analysis.

Ray field maps in the position/angle domain are expected to be useful in various other applications of ray theory besides imaging. Within certain practical limits the maps may contain information on every ray in the medium that intersects the acquisition surface. This amount of information may, for example, be utilised by extensions of ray theory such as Maslov, Gaussian beams, and coherent state methods to obtain wave field solutions with a wider range of validity than zerothorder ray theory.

Also, the one-to-one mapping between position/angle coordinates and ray field coordinates may be exploited by making a change of variables in practical calculations. As demonstrated in Appendices A and B, this may be advantageous in tomography and the forward calculation of ray fields maps directly on a grid in the position/angle domain. These possibilities are interesting areas for future research.

7.4.2 Algorithms

Adaptations of two well-known ray tracing algorithms to the position/angle domain have been studied. In Chapter 5 it was concluded that the ray field construction algorithm in its current form is not suited for application in the position/angle domain. The main problem is the accommodation of deformations in the ray field structure during propagation. It was found that in the position/angle domain this deformation is predominantly shear-like, whereas the algorithm is designed primarily for diverging ray fields.

In fact, from the findings of Chapter 6 it may be concluded that the ray field construction algorithm is actually overqualified for application in the position/angle domain. The geometrical spreading of ray fields is limited in the position/angle domain, and the corresponding ray field maps are uniformly single-valued. Therefore, two of the most important reasons for developing wave front construction methods in the spatial domain, viz. the occurrence of shadow zones and multi-pathing, are absent in the position/angle domain. As a result, the more primitive but also more efficient paraxial ray methods were adapted successfully to the position/angle domain. In Chapter 6 the paraxial ray method was shown to work adequately in the example provided. The gain in speed with respect to tracing upwards is roughly proportional to the average ray length divided by the spatial grid interval.

Stop press: upon submission of the manuscript of this thesis the author was made aware of the paper by Fomel and Sethian (2002), who propose an algorithm that, basically, has the same objective as the work described here. They also consider an enlarged domain of position and angles, in which they determine what they call the "exit times and positions".

In fact, the algorithm they propose is very similar to the one that was presented in Kraaijpoel et al. (2002), which was not included in this thesis. The essence of both of these algorithms is to trace short ray segments up to a point where the relevant information is available and may be interpolated. The general disadvantage of this approach is the repeated interpolation, which makes the algorithms very sensitive to error propagation. In this respect, the advantage of the paraxial ray method of Chapter 6 is that – apart from the errors in ray tracing that are present in all approaches – an error is introduced only once, in the evaluation of the averaging integral, and hence error propagation is not an issue. Moreover, Fomel and Sethian (2002) require a sorting procedure to determine the order in which the grid points should be updated. In the paraxial ray method this sorting is implicitly performed by the propagating rays. Nevertheless, a quantitative comparison will be an interesting objective for future research.

Finally, in Appendix B a set of equations is presented for the calculation of ray fields maps directly on a grid in the position/angle domain. In theory, this approach may lead to a very efficient algorithm, because it avoids two computationally expensive steps that are common to the other ray methods: first, the explicit mapping from spatial (or position/angle) coordinates to ray field coordinates, and second, the (sufficiently smooth) interpolation of medium properties at arbitrary spatial locations.