Chapter 7

Conclusions and outlook

In this thesis we have presented the derivation of an effective quantum field theory suitable for the description of a Bose gas near a Feshbach resonance. We have presented several applications of this theory, both above and below the critical temperature for Bose-Einstein condensation. In the last part of this thesis we have studied in detail the magnetic-field dependence of the frequency of the coherent atom-molecule oscillations and have obtained excellent agreement with the experimental results. In particular, we have been able to quantitatively explain the many-body effects on this frequency by making use of a linear response approximation to our mean-field equations. Although we have already presented some numerical solutions of the mean-field equations that improve on this approximation, a great deal of work still has to be done. The numerical solution of these equations for the situation of time-dependent detuning is rather involved. Nevertheless, work in this direction is in progress and will be reported in a future publication.

As already mentioned, we have also discussed the properties of the gas above the critical temperature. This discussion was mainly concerned with the equilibrium properties of the gas and we studied the many-body effects on the bound-state energy of the molecular state. An important conclusion of this study is that, for certain values of the parameters, there exists a many-body induced resonant state with a relatively small energy. In future work we intend to study the effects of the appearance of this resonant state in the molecular density of states on the properties of the gas. In particular we expect that due to this effect the number of molecules in the gas will be large even at relatively small detuning, which can not be explained on the basis of two-atom physics.

Furthermore, to study the normal state also in an out-of-equilibrium situation, we should derive a quantum kinetic theory that describes the evolution of the local occupation numbers of the atoms and molecules. Moreover, the description of the Bose-Einstein condensed phase of the gas at nonzero temperatures requires a modification of the mean-field equations such that they include the effects of the thermal clouds of atoms and molecules, and we need equations for the evolution of the local occupation numbers of the latter. The extension of the theory presented in this thesis to these situations can be derived in a unifying manner by using a functional formulation of the Schwinger-Keldysh nonequilibrium theory [104], and is especially important in view of the ongoing effort to produce ultracold molecules by means of a sweep in the magnetic field through the Feshbach resonance [62].

The theory presented in this thesis is generalized to a gas of fermionic atoms in a straightforward manner [63, 80]. One modification is that to have s-wave scattering between fermionic atoms we have to have a mixture of atoms with two hyperfine states, since the Pauli principle forbids s-wave scattering between identical fermions. Furthermore, the properties of the dressed molecular state is altered due to the presence of the Fermi sphere. A molecule with zero momentum only decays if its energy is above twice the Fermi energy. If the molecular state lies below twice the Fermi energy, the equilibrium situation is a Bose-Einstein condensate of molecules. If we start from this situation and increase the detuning, the Bose-Einstein condensate of molecules crosses over to a Bose-Einstein condensate of Cooper pairs, i.e., a BCS-BEC crossover occurs [57, 61]. We intend to study this crossover, and in particular the behaviour of the critical temperature, in detail in future work.

Clearly, Feshbach resonances present an exciting opportunity for the experimental and theoretical study of the many-body properties of atomic and molecular Bose and Fermi gases. There is little doubt that these Feshbach resonances will find many new applications in the years to come.