
Table of Contents

1	Introduction	1
1.1	Statistics education	1
1.2	Design research	3
1.3	Symbolizing	3
1.4	Computer tools	4
2	Background and research questions	5
2.1	Realistic Mathematics Education (RME)	5
2.2	Trends in statistics education research	8
2.3	Nashville research with the Minitools	17
2.4	Research questions	34
3	Methodology and subjects	37
3.1	Design research methodology	37
3.2	Hypothetical learning trajectory (HLT)	39
3.3	Phase 1: Preparation and design	41
3.4	Phase 2: Teaching experiment	42
3.5	Phase 3: Retrospective analysis	45
3.6	Reliability and validity	46
3.7	Overview of the teaching experiments and subjects	47
4	A historical phenomenology	51
4.1	Purpose	51
4.2	Method	52
4.3	Average	53
4.4	Sampling	61
4.5	Median	65
4.6	Distribution	74
4.7	Graphs	80
4.8	Summary	87

5 Exploratory interviews and a didactical phenomenology 91

5.1	Exploratory interviews	92
5.2	Didactical phenomenology of distribution	100
5.3	Didactical phenomenology of center, spread, and sampling	104
5.4	Initial outline of a hypothetical learning trajectory	109

6 Designing a hypothetical learning trajectory for grade 7 111

6.1	Outline of the hypothetical learning trajectory revisited	111
6.2	Estimation of a total number with an average	112
6.3	Estimation of a number from a total	114
6.4	Talking through the data creation process	114
6.5	Data analyst role	115
6.6	Compensation strategy for the mean	117
6.7	Data invention in the battery context	121
6.8	Towards sampling: Trial of the Pyx	123
6.9	Median and outliers	124
6.10	Low, average, and high values	126
6.11	Reasoning about shape	127
6.12	Revision of the Minitools	133
6.13	Is Minitool 1 necessary?	134
6.14	Reflection on the results	136

7 Testing the hypothetical learning trajectory in grade 7 141

7.1	Pretest	142
7.2	Average box in elephant estimation	145
7.3	Reliability of battery brands	148
7.4	Compensation strategy for the mean	150
7.5	Students' notions of spread in the battery context	151
7.6	Data invention	156
7.7	Estimating the mean with the median	157
7.8	Average and sampling in balloon context	160
7.9	towards shape by growing a sample	161
7.10	Average and spread in speed sign activity	166
7.11	Creating plots with small or large spread	168
7.12	Jeans activity	170
7.13	Final test	171
7.14	Answer to the first research question	178

8	Diagrammatic reasoning with the ‘bump’	187
8.1	From chains of signification to Peirce’s semiotics	187
8.2	Semiotic terminology of Peirce	190
8.3	Analysis of students’ reasoning with the bump	199
8.4	Answer to the second research question	205
9	Diagrammatic reasoning about growing samples	211
9.1	Information about the teaching experiment in grade 8	211
9.2	Larger samples in the battery context	214
9.3	Growing a sample in the weight context	218
9.4	Reasoning about shapes in the weight context	225
9.5	Growing the jeans data set in Minitool 2	231
9.6	Growing samples from lists of numbers	233
9.7	Final interviews	235
9.8	Answer to the integrated research question	239
10	Conclusions and discussion	243
10.1	Answers to the research questions	243
10.2	Other elements of an instruction theory	256
10.3	Discussion	265
10.4	Towards a new statistics curriculum	273
10.5	Recommendations for teaching, design, and research	276
	Appendix	283
	References	285
	Samenvatting	301
	Curriculum vitae	313
