Chapter 3

Scholarship and rainwater: Eratosthenes Batavus

3.1 Introduction

In the years between circa 1614 and 1625, Snellius exerted himself to determine the length of the circumference of the earth. He published the major part of his results in his *Eratosthenes Batavus* of 1617. This is one of the key works on which Snellius's posthumous fame is based. One of the striking aspects of the book is its diversity. It offers a survey of old knowledge, a methodological improvement of surveying, practical considerations about taking measurements and a load of data. Snellius was the first to measure the circumference of the earth on the basis of a triangulation, that is, the plotting of a net of triangles between two towns far apart.[1]

This interdisciplinary, time-consuming operation must have demanded much of Snellius's attention. Both this fact and the rich contents of the *Eratosthenes Batavus* make it a central work in Snellius's oeuvre. The book will be discussed in this chapter, together with Snellius's work in the same field done after the publication of the book.

The aim of this chapter is threefold. In the first place, a concise overview of Snellius's method will be given, with special attention to those aspects to which he himself drew the attention. This will show, among other things, a side of Snellius unknown in secondary literature: he was an experimenter with some knowledge of chemistry. In the second place, an attempt has been made to carefully establish the chronology of Snellius's activities relating to *Eratosthenes*

[1][Haasbroek, 1968, p. 63]. For a summary of the book, see [Wolf, 1973b, pp. 170–174] and for a more elaborate survey the first chapter of [van der Plaats, 1889].

Batavus, which adds to his biography. In the third place, the results of this chapter will be used for the general analysis of Snellius's mathematics in chapter 8.

3.2 Purpose and method of Eratosthenes Batavus: general use versus endless efforts

The task of measuring the earth was a huge one, and it is evident that Snellius would not have ventured on this enterprise if he had not perceived it as very useful. He gave several motivations in his dedicatory letter to the States General. According to Snellius, the question of the size of the earth was a very old one, which had occupied many scientists. Hence the person who answered it would make a good addition to scientific knowledge and could make a claim to eternal fame.

Besides, the problem of determining one's longitude was most urgent, especially for the Dutch ships which ventured far away from home to formerly unknown regions. Snellius proudly proclaimed his contribution to the solution of this problem:

> I have tackled a problem the solution of which has always been desired by everyone, which has been tried very often, and which has also been made famous by the endeavours of great men. I present here an accurate assessment of the size of the globe [...].[2]

Indeed, the circumference of the earth was a relevant parameter in some methods for determining one's position at sea. These estimated the distance sailed by the ship on the basis of its velocity, and used it to calculate its new position when its original position was known.[3] Linear distance (travelled miles) was transformed into angular distance (the angle between a referential meridian and the present meridian) in this way. Other methods only used this angular distance, for instance those based on astronomical observations or time differences between the point of departure and the meridian under consideration.

In fact, the *Eratosthenes Batavus* offers no direct link between the meridian measurement and the problem of finding longitude at sea. Snellius only seems to have mentioned the problem to explain the relevance of the book to the dedicatees. And these indeed invited him a few years later to become a member of a committee installed by them to judge an alleged solution to the problem. The

[2]'Rem aggressi sumus ab omnibus semper desideratam, saepius tentatam, et magnorum quoque virorum industria nobilitatam [...] Orbis terrae quantitatem accurate definitam hic exhibeo, ut inde omnis longitudinis et latitudinis mensura ex itinerum intercapedine tanto minus erroribus sit obnoxia.' [Snellius, 1617b, fol.)?(*iii*^{v}–)?(*iiii*^{r}].

[3]Cp. Jarich's method in section 2.9.1.

problem again occupied him in his book *Tiphys Batavus*. The States General had offered an opulent financial recompense for the solution of this problem.

Rudolph Snellius also served as an examiner of several proposed solutions. The inventor of one of them, Thomas Leamer, was so disappointed when his method was not accepted that he devoted a long pamphlet to the explanation of his method and the refutation of the objections of Rudolph Snellius and Robbert Robbertsz. One of the key elements of Leamer's method consisted of assigning numerical values to Hebrew words in order to discover their hidden meanings. Snellius and Robbertsz also made Leamer calculate some examplary problems, but they found the results unsatisfying and concluded that he did not even master the foundations of astronomy.[4] This polemic may have induced father and son Snellius to discuss more sensible approaches to the longitude problem, and it may have led Willebrord Snellius to examine one aspect profoundly.

Furthermore, the work had a more local interest: Snellius surveyed a large part of Holland and the surrounding provinces which enabled the States General 'beyond doubt, to register their home-country more accurately' than the Greeks, Romans or any other rulers.[5] Altogether, the problem was challenging for a scholar and its solution well-suited to serve the public good. Moreover, it was useful for astronomy, which was probably not mentioned in the dedicatory letter because this application was less relevant for the dedicatees. The size of the earth was an important parameter in some astronomical calculations, e.g. of the solar distance. This consideration must also have stimulated Snellius.

Snellius's project can be divided into a number of steps, which will first be mentioned briefly and then explained more elaborately. The main source is the *Eratosthenes Batavus*. Yet after its publication, Snellius continued his measurements, because he was not satisfied with his results. Part of his corrections and additions, some of them several pages long, can be found in his own copy of the *Eratosthenes Batavus*, in which he prepared a second edition that has never been published. This copy is now in the Royal Library in Brussels. Most of these changes were published in 1729 by Petrus van Musschenbroek in his *Physicae experimentales, et geometricae, ... de magnitudine terrae ... dissertationes* ('Physical-experimental and geometrical discourses on the size of the earth'). The manuscript which Van Musschenbroek had at his disposal was slightly different from the Brussels copy, containing more changes in some places and fewer in other.

Snellius's measurements and calculations, including those from after the publication of *Eratosthenes Batavus*, have been studied thoroughly by N.D. Haasbroek, lecturer at the department of surveying of the Technological University of Delft. He also compared Snellius's results with modern data, made his own

[4][Leamer, 1612], cp. [de Waard, 1912b].

[5]'Patriam autem hanc iam accuratius et certius consignari posse ex ipso opere facile constabit.' [Snellius, 1617b, fol.)?(*iiii*^{r}].

calculations to check those of Snellius and scrutinized Van Musschenbroek's additions. Part of what is written below is founded on Haasbroek's excellent work, published in four Dutch articles and in a book in English. For technical details I refer to these publications, where a number of relevant maps can be found as well. The focus of the current chapter is somewhat different: it is less technical, and more methodological and historical than Haasbroek's.[6]

Snellius applied a *triangulation*, that is, a method to survey land by dividing it into triangles. He improved earlier efforts by Gemma Frisius and Tycho Brahe. Gemma Frisius explained the principles of triangulation for the first time in an appendix to his *Cosmographicus liber Petri Apiani*, published in 1533. The surveyor was to collect the data of the directions of different places from one place by means of a magnetic compass and a large circle, then travel to the next place and repeat the procedures. The distances between these places could be determined by walking and counting the steps. Later in the book, Gemma Frisius proposed to take the angles of the network instead of the directions, draw them on a map and calculate the required distances by using proportions. Thus no trigonometrical functions were used. Snellius would improve the precision of the method by calculating the sides of the triangles in the network by means of trigonometrical functions instead of measuring them on a map. There are no indications that Frisius actually carried out a substantial triangulation.[7]

The direct inspiration for Snellius's endeavours was probably Tycho Brahe, who performed a triangulation in Denmark. Snellius knew Tycho personally. The latter used a combination of astronomical observations (azimuths, see below) and angle measurements to interrelate the positions of a number of Danish localities. However, he did not actually calculate these positions. If he had, he would have noticed that his results were not very accurate.[8]

Snellius's programme for the determination of the circumference of the earth consisted of the following steps, in the order of his own presentation:[9]

1. He studied the works of classical and early modern authors on the same issue,
2. and he defined his unit of length carefully.
3. He measured several base lines in the fields around Leiden.

[6]Cp. the introduction to this chapter. [Haasbroek, 1960], [Haasbroek, 1965], [Haasbroek, 1966], [Haasbroek, 1967]; the part on Snellius of [Haasbroek, 1968] has almost the same content as the Dutch articles.

[7][Haasbroek, 1968, p. 7, 10–14].

[8]For Snellius's visit to Tycho see section 2.7. An elaborate study of Tycho's triangulation is in [Haasbroek, 1968, pp. 29–58].

[9]Cp. [Haasbroek, 1968, p. 66]. Because Snellius did not publish his later emendations himself, his own 'presentation' does not apply there; that part is based on chronology as much as possible.

4. He calculated the distance between Leiden and The Hague.
5. From a number of towers in Dutch towns, he measured the directions towards other towers. He also measured another base line.
6. He calculated the distances between these towns and their relative position.
7. As a result, he could determine the distance between Alkmaar and Bergen op Zoom, two places with a small difference in longitude.
8. He determined the latitude of Leiden, Alkmaar and Bergen op Zoom, and the azimuth from Leiden to The Hague.
9. He used all these to calculate the circumference of the earth and published his work in *Eratosthenes Batavus* (1617).
10. After the publication of the book, he repeated many of the measurements and calculations.
11. In 1622 he measured a new base line, but he did not make his calculations anew,
12. and he extended his triangulation network to the Southern Netherlands.
13. He collected new material through (among others) Gassendi.

3.3 Before publication: a scholar dirties his hands

Snellius did most of his work before the publication of the *Eratosthenes Batavus*. Afterwards, he did not make any fundamental changes. His work will be discussed by elaborating on the steps given above.

Ad 1. The *Eratosthenes Batavus* consists of two books. Its first book is devoted to a historical survey. Snellius shows himself a true humanist scholar here, knowing an extended range of sources and able to use them. He addressed a number of relevant issues, such as the shape of the earth, its location in the universe,[10] and earlier endeavours to measure the earth.

Snellius's most famous predecessor was Eratosthenes of Cyrene (276–194 BC), who had computed the circumference of the earth on the basis of the distance and the difference in latitude between two localities in Egypt almost on the same meridian. Snellius devoted many pages to a precise explanation of this work, including many figures and calculations. He also had some critique—understandably: if Eratosthenes's work had been perfect, there would have been

[10] Cp. section 4.5.

no need for his own enterprise. Snellius also mentioned Hipparchus and Ptolemy and discussed the work of some Arab scholars and of Jean Fernel, a sixteenth-century French author, as well.

Ad 2. Snellius was aware that it only made sense to talk about distances if his readers knew exactly what unit of measurement had been used. As there existed no standard units of measurement, this was no trivial problem. The first part of the second book was devoted to its solution. He developed no less than four means to convey his information. The first of them was to relate the unit used by him, the Rhenish foot ('pes Rijnlandicus', 'Rijnlandse voet'), to Roman measures, for which the ancient sources were scrutinized again. These were not only bookish sources, but also archeological evidence: he considered the dimensions of the Brittenburg, a Roman fortress near Katwijk (and Leiden). His conclusion was that the Roman and the Rhenish foot were equal.[11]

The most direct way to inform his readers about the used unit of measurement was to show it to them. This was somewhat problematic, however, because the size of the paper changed in the process of printing, as Snellius explained to the reader. Even so, he included a picture of half a Rhenish foot in the book. After printing, this turned out to have the wrong measure, which necessitated a correction on the last page of the book.[12]

Snellius also expressed the length of the Rhenish foot by relating it to other Dutch and foreign standards. Firstly, Snellius gave this information for different feet. He had investigated this himself in Dordrecht, Den Briel, Middelburg, Goes, Zierikzee, Antwerp, Leuven and Mechlin, and the information had been sent to him from other places. He had also borrowed some information from books. One of them had been sent to him by the Bohemian Joannes Smil a Michalovicz, someone he probably knew from his stay with Tycho Brahe.[13] This shows Snellius's methods of acquiring knowledge in a nutshell: he used printed sources, acquaintances and his own observations. It also tells us that he had travelled to even more places than those in his triangulation network, which did not include Zeeland, Den Briel or Leuven.

Secondly, Snellius related the Rhenish foot to ells. Snellius explained that the correct (local) measure of the ell was made known publicly in towns to prevent tradesmen from fraud, because this unit of measurement was used for cloth. He had gained information in the same three ways as for the foot, now giving first hand information for Oudewater, Leiden, Amsterdam and Antwerp.[14]

Snellius then devoted an interesting excursus to his highly original fourth endeavour to fix the value of the Rhenish foot. He argued that although units of length were not well established in general, this problem did not hold for weights,

[11] [Snellius, 1617b, p. 132].
[12] [Snellius, 1617b, pp. 124, 194, 264].
[13] [Snellius, 1617b, pp. 124–126].
[14] [Snellius, 1617b, pp. 140–143].

because these were connected directly to the value of coins and therefore were supervised continuously. He first gave an overview of the weights and relative values of a number of coins. Then he announced that he would determine the weight of a cubic Rhenish foot of water accurately, aiming posterity to reap the fruits of his labour.

A careful description of his experiment was necessary,

> because this is a topic that has kept men of learning fully engaged, and in which some have too boldly dared to determine something just on the basis of their own thoughts.[15]

He needed to measure the volume and the mass of an amount of water exactly. While the determination of the mass was not difficult, the determination of volume was more troublesome, and Snellius had to design a special measuring instrument to cope with some of the complications. First, he had a hollow cylinder of copper or bronze made with a diameter of half a foot and equal height. He made a hole in its upper covering to which he fitted a smaller cylinder (diameter and height 0.1 foot). The insides of the two cylinders were now connected. He also made a small outlet in the covering of the larger one to let the air escape, to enable it to be filled with water completely. After having done so, he closed this opening with a screw and filled the small cylinder.

Snellius wrote that he had constructed the instrument in this way to master the 'rising' or 'swelling' (*tumor*) of the water, which had troubled him at first. He probably referred to the phenomenon that the surface of water in a vessel is not flat due to the force of adhesion. The construction with the two cylinders seems to have been designed to ensure that the instrument both had a rather large volume and that it could be filled completely, so that he could determine the volume of the water exactly. This also explains why he did not take a ready-made instrument for measuring the volume, e.g. a jug: its contents could never be determined with the required degree of precision. After filling, he connected three screws to the bottom to make the vessel stand horizontally.

Now that the instrument was practicable to measure precisely the volume of water, he had to consider what sort of water he could employ. In the beginning he had decided to use rainwater, assuming that rainwater would be equally heavy to all experimentators everywhere because it came straight from the heavens. He compared the weight of rainwater and well water on 10 May 1617. After discovering to his dismay that the former was heavier than the latter, he understood that this could happen because it had rained very recently, and thus the water had arrived in the water reservoir carrying along dregs which had not had the time to settle down.

[15]'cum ista quoque materia sit quae viros doctos admodum habuit sollicitos, et in qua non nulli nimis audacter ex suo tantum conceptu quidquam definire ausi fuerint.' [Snellius, 1617b, p. 150].

Snellius then decided to use a chemical process as an auxiliary means to solve this problem. He distilled the rainwater to purify it, for which he employed a bain-marie (*balneum Mariae*), because he considered that to be the least violent method of all. To do so, he filled a cooking-pot with water, put a vessel of glass half-full of the rainwater in it and put another vessel on top. When he had brought the water in the cooking-pot to the boil, the rainwater also started to evaporate. Through the upper vessel the steam was directed to a tube from which it emerged as purified water. Next, he weighed the double cylinder twice: once empty and once full of distilled water. On 31 May, he filled it again with rainwater from the cistern, after a period of 10 or 12 days during which the sediments had settled down. He reported its weight, and also the weight of the instrument filled with well water.

He subsequently found the weights of the three kinds of water by subtracting the weight of the vessel from the totals. He used his knowledge of the volumes of bodies to calculate the weights of the different kinds of water in the large cylinder, which was easy because of the similarity of the large and small cylinder. He remarked that the weights were in the same proportion as the volumes and therefore the weight of the water in the total vessel was to the weight of the water in the large cylinder as $0.5^3 + 0.1^3$ to 0.5^3, that is 126 : 125. The weight of the water in a cylinder with diameter and height 1 foot would be 8 times that of the water in the big cylinder.

The calculation of the weight of a cubic Rhenish foot of water was equally straightforward: the proportion between the weight of the water in a cylinder with diameter and height 1 foot and a cubic foot of water was $\pi : 4$. Snellius gave the final values and concluded optimistically that his data made it easy for his readers to determine the size of the Rhenish foot expressed in their own unit of length, thus ignoring all his own troubles.[16]

Snellius's experiment has gone unnoticed in modern Snellius scholarship,[17] perhaps because it does not fit in the modern picture of the activities which a mathematician should pursue. Yet it is full of information, stirs the imagination and raises many intriguing questions. Although his description is very detailed, some problems were glossed over. Did he consider taking other material than water, with a weight that would depend less on the circumstances? Did he consider the change of volume of the water due to temperature shifts? Was the vessel not deformed when filled with water? Snellius's description of his instrument and experiment are so detailed that in principle other scientists could repeat and check it if they wanted, but the many complications with which he was confronted make one wonder whether this was actually possible. Hence although Snellius described many practical details, the experiment seems to be mainly relevant as a thought experiment: how a unit of length can in principle

[16] [Snellius, 1617b, pp. 143–156].

[17] I have only found a short summary in [van der Plaats, 1889, p. 7].

be expressed in a unit of weight. Snellius could also use his description of the experiment as a showcase, showing his aptness in solving practical problems and some knowledge of chemistry. It is unknown whether he ever did similar experiments and where he had acquired the necessary knowledge.

One reaction to the experiment is known. In a letter to Snellius, Gassendi discussed the experiments which Joannes Lombardus had done to establish the weight of solid palms of water, wine and oil. Gassendi used Snellius's relative density of water to compare Lombardus's palm to Snellius's foot, unfortunately having to conclude that the result did not match the relation between those two units as known from elsewhere. He did not know where this difference came from. Snellius shared this interest in the precise determination of standard measures along with some other contemporary scholars.[18]

Even though Snellius gave all this information, it cannot be made out exactly how long the Rhenish rod ('Rijnlandse roede') used by Snellius was. Haasbroek used 1 rod = 3.766 m. Although traditionally the rod was divided into 12 feet and a foot into 12 inches, Snellius used a decimal division to facilitate calculations (thus, 1 foot = 0.3766 m).[19] This choice may well have been influenced by Stevin's plea for decimalization.

Ad 3. Snellius used a surveyor's chain to measure three base lines, two in the fields between Leiden and Zoeterwoude (one on the straight line between the towers of the two places and one perpendicular to it) and one between Wassenaar and Voorschoten (all three villages in the neighbourhood of Leiden). The first one is the shortest: 87.05 rods (327.8 m). By measuring angles between the lines connecting the end points of the base lines and points in the two places nearby, he could calculate the distance between these points in Leiden and Zoeterwoude, and between those in Wassenaar and Voorschoten.[20]

Ad 4. Although all the localities under consideration were located on a sphere (the earth), Snellius did not use spherical trigonometry, but calculated as though all places were in one plane. He was aware that this was a simplification, but because the distances were not very large, the differences in his results were negligible. This means that he adjusted his degree of exactness to his aims in practical geometry.[21]

After having done his first series of measurements in the fields, he conducted the rest of his observations from towers, in this way having a higher viewpoint, which enabled him to oversee larger distances. He did some measurements from the Town Hall in Leiden and from a church in The Hague and used them, together with his earlier results, to calculate the distance Leiden–The Hague in

[18] [Gassendi, 1964, p. 7], [Gassendi (S. Taussig ed.), 2004, 2, pp. 13–15].
[19] [Snellius, 1617b, p. 157], [Haasbroek, 1968, pp. 63–65].
[20] [Snellius, 1617b, pp. 156–160, 163–164], [Haasbroek, 1968, p. 70].
[21] [Snellius, 1617b, p. 198].

two different ways.[22]

Ad 5. Snellius then established a triangulation network connecting Alkmaar and Bergen op Zoom. This was a schematic representation of a number of Dutch towns, interconnected by straight lines representing their distances. Snellius did not plot them on a map, but only made a sketch. This did not have to be to scale because it was not meant for measuring: the distances were calculated on the basis of the measurements of the angular distances each time between two places, starting from the base lines.

The actual determination of the distances in the triangulation network was a huge work, which involved much travelling. Snellius did observations in Leiden, Alkmaar, Haarlem, Amsterdam, Utrecht, Gouda, Oudewater, The Hague, Zaltbommel, Breda, Willemstad, Dordrecht and Bergen op Zoom. He took his measuring instruments to all these places and climbed towers, from where he measured the angular distances between the towers in other places of his network. Rotterdam was also included in the network, but no measurements were taken from there.[23]

He did not have to do all these observations by himself. In 1615, he travelled around with Erasmus and Casparus Sterrenberg, two young barons. Snellius told the readers of the *Eratosthenes Batavus* that they had already learned arithmetic, geometry and (spherical) trigonometry and now liked to exercise their abilities in a useful matter. When they longed to relax their minds somewhat during the summer holidays, their tutor Joannes Philemon proposed to them to travel to the adjacent regions to prevent his pupils from dissipating their time in too much leisure. They asked Snellius to come along, stimulating him to try his hand at determining the circumference of the earth, 'which I had once said in passing but which they took in earnest'.[24] Their role must have been somewhat exaggerated here.

The Sterrenbergs did part of the observations and calculations. The group first travelled to Oudewater, where Snellius's widowed mother lived. Snellius described his father's place of birth elaborately in the *Eratosthenes Batavus*, dwelling on the horrible siege of 1575, the cruelties done by the Spaniards after their victory and the demolition which it had suffered some decades earlier. Near the town, Snellius and his companions measured a new base line, which they used to determine the distance between Oudewater and Montfoort.[25] In August, they travelled hence to Amsterdam. In the same year, Snellius did a stellar observation in Mechlin together with the Sterrenbergs and Philemon, in order to determine the geographical latitude. He dedicated the second book of

[22] [Snellius, 1617b, pp. 161–167].
[23] [Haasbroek, 1968, p. 88] gives a table and a schematic map of all the 54 measured angles.
[24] 'Ecce quid facerem illa quae olim tantum obiter a me dicta, serio ab ipsis accepta.' [Snellius, 1617b, p. 177].
[25] [Snellius, 1617b, pp. 176–179].

the *Eratosthenes Batavus* to them.[26]

The taking of the measurements was a time-consuming business, which took over a year. An indication of this is that Snellius determined the distance between Wassenaar and Voorschoten twice, based on two different configurations, the measurements for which were done more than a year apart, according to Snellius's own testimony. He added that they had both been witnessed by knowledgeable people, and in one of the measurements he had also been helped.[27]

Snellius was hampered by all sorts of practical difficulties, some of which he disclosed to his readers. When standing on towers, he was bothered by the wind, which was stronger than on the ground. The observations were also made less accurate by the fact that it was often not possible to stand at the centre of the tower. In addition, it could be difficult to identify the correct towers from a large distance.[28]

He stressed these difficulties to impress the reader by his Herculean task, which he would have abandoned

> if the general profit, and the illustrious energy invested on this topic in so many centuries before me, had not spurred me on and forced me to take my pen in hand again, and to raise my body and the sharpness of my eyes to the peaks of towers.[29]

And he had told the reader only the tip of the iceberg:

> What I report here is hardly the hundredth part of the exertion, trouble and expenses which I have endured.[30]

To emphasize this rhetorical exaggeration, he even proposed that they replace 'one hundredth' by 'one thousandth' in the second edition of *Eratosthenes Batavus*. He clearly wanted to ascertain that the reader would appreciate his *magnum opus* sufficiently.

[26]The data about the year of the travel are not consistent. Observation in Mechlin in 1615: [Snellius, 1617c, ad p. 208-1] (cp. nr. 12 of Snellius's programme). In the dedicatory letter to the Sterrenbergs, Snellius refers to their summer holiday trip two years before, which should be 1615—unless the letter was written some months before the publication of the book, then it could be 1614, [Snellius, 1617b, pp. 119–120]. In a note to Risnerus's *Optica*, Snellius wrote that he travelled from Oudewater to Amsterdam in 1615 with the Sterrenbergs. However, in *Eratosthenes Batavus* he wrote that they were in Oudewater in the year after the death of Rudolph Snellius (1613) ('[...] charissimi parentis mei obitum anno superiore [...]', [Snellius, 1617b, p. 177]).

[27][Snellius, 1617b, p. 164].

[28][Snellius, 1617b, p. 171].

[29]'nisi publica utilitas, et tot iam seculis fatigata tam nobilis cura stimulos mihi addidisset, et rursum calamum in manum, corpus et oculorum aciem in turrium fastigia attollere coëgisset.' [Snellius, 1617b, p. 171].

[30]'Haec enim ipsa quae hic affero vix centesima pars sunt laboris, molestiae, impensarum quas exantlavimus.' [Snellius, 1617b, p. 171].

When Snellius made his measurements for the *Eratosthenes Batavus*, he needed instruments that were both accurate and could survive transport. On his expedition with the Sterrenbergs, he used a semi-circle (diameter $3\frac{1}{2}$ feet, about 1.33 m) for measuring angular distance between towers (in the horizontal plane) and a quadrant of iron with a radius of over $5\frac{1}{2}$ feet (a. 2.09 m) with an edge of copper for determining the polar altitudes. He used a smaller quadrant ($2\frac{1}{5}$ feet, a. 84 cm) when determining the position of his base line between Leiden and Zoeterwoude. When he was in Gouda, he measured the angle Leiden-Gouda-Den Haag several times, both with this last quadrant and with his semi-circle.[31]

Snellius received some data from other scholars. Beeckman for instance measured the angular distances between a number of places from the tower of Zierikzee on Snellius's request, yet Snellius did not include them in the *Eratosthenes Batavus*.[32]

Haasbroek pointed out some deficiencies in the triangulation network, e.g. the insufficient determination of the positions of Haarlem and Amsterdam in relation to the other points in the network.[33]

Ad 6. On the basis of the measured angles and the length of one side of the network (Leiden–The Hague), the directions and the lengths of all other sides could be calculated by simple trigonometry. Snellius did not manage to make all these calculations without making errors, as has been remarked by Van Musschenbroek and Haasbroek. The latter called him 'a shoddy calculator' and noticed that Snellius did not check all his calculations, although he sometimes could have done so by computing the same distance in different triangles.[34] Of course, he did not have a method like that of least squares at his disposal to minimize the influence of measuring errors, but he corrected the outcomes of his calculations in such a way that the sum of the angles of every triangle was 180^0.[35]

Ad 7. Snellius used his previous work to calculate the distance between

[31] '[...] semicirculum diametri pedum Rhijnlandicorum trium et semis, ad gaeodaesias distantiarum et angulorum quantitatem e turribus observandam. Quadrantem etiam amplissimum ferreum, aere incrustatum, amplius quinque et semis pedum, ad poli altitudinem explorandam.' [Snellius, 1617b, p. 177], '[...] maximo quadrante ferreo cuius radius propemodum erat sexpedalis, limbus autem ad commodiorem sectionem aere incrustatus [...]', [Snellius, 1617c, ad p. 208-1] (edited in [Bosmans, 1900, p. 121]); cp. [Haasbroek, 1968, p. 65] for an English translation of p. 177. 'Aes' meant any crude metal except gold and silver, esp. copper, or an alloy, mainly bronze (probably not brass, which is 'orichalcum', a word indeed used elsewhere by Snellius, [Snellius, 1617b, p. 150]). The quadrant of $2\frac{1}{5}$ feet was 'aereus', which can mean made of copper, or furnished/covered with copper/bronze, [Snellius, 1617b, p. 156].

Measurement from Gouda: [Snellius, 1617b, pp. 167–169].

[32] [de Waard, 1939, p. 105]. Of the places mentioned, only Bergen op Zoom was in Snellius's network.

[33] [Haasbroek, 1968, pp. 99-100].

[34] [van Musschenbroek, 1729, pp. 359–380], [Haasbroek, 1968, pp. 72–73].

[35] [Haasbroek, 1968, p. 89].

Alkmaar and Bergen op Zoom. His final value is 34,710.6 rods (130.7201 km), which differs about 172.5 m from the same value according to modern measures.[36] Haasbroek concluded more mildly than in the previous section:

> It is the best obtainable result in those days, also thanks to the eminent determination of the base line *LHg* [sc. Leiden–The Hague] and its excellent checks and in spite of the rather poor construction of the northern part of the meridian chain and the many errors in the calculation.[37]

Ad 8. Snellius then had to find the geographical latitudes of Alkmaar, Bergen op Zoom and Leiden, which he did by measuring the height of the Pole Star. His instruments did not allow him to determine these latitudes with the same degree of precision as the distance between Alkmaar and Bergen op Zoom. He also determined the azimuth (the direction in relation to the meridian) from his own house to the Leiden town hall and to The Hague and used it to calculate the azimuth Leiden–The Hague. This was necessary to orient his triangle network and thus to determine the difference in longitude between Alkmaar and Bergen op Zoom. He had selected these last two places because they were located almost on the same meridian.[38]

In connection to this, Snellius solved a geometrical problem, which gave him some lasting fame. It is called the Resection Problem. Snellius had to determine the distance of his house to three points in Leiden (two churches, the Pieterskerk and Hooglandse Kerk, and the Town Hall), the mutual positions of which were known. He considered his solution of the problem to be of no little importance himself: he devoted a separate, rounded-off chapter to it and proudly announced his useful invention for surveying:

> I have invented an elegant theorem for that problem, which can have a widespread application in our country from now on, because the distances between so many illustrious places have been registered with such precision.[39]

From a geometrical point of view, the problem is no more complicated than many other construction problems. Its practical application, however, made it stand out. Snellius's solution consisted of five steps:[40]

[36][Snellius, 1617b, p. 194], [Haasbroek, 1968, pp. 104–105].

[37][Haasbroek, 1968, p. 105].

[38]Cp. [Haasbroek, 1968, pp. 105–107].

[39]'Et ad eam rem theorema scitum excogitavi, cuius usus iam in patria nostra deinceps permagnus esse possit, cum tot illustrium locorum intervalla tam accurate sint consignata.' [Snellius, 1617b, p. 199].

[40][Snellius, 1617b, pp. 203–206].

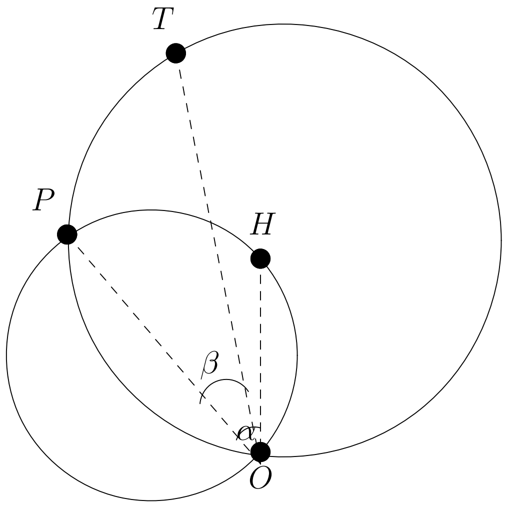

Figure 3.1: The Resection Problem

1. Two more data were needed to determine the problem. A number of possibilities existed for this, but as this is a problem from applied geometry, Snellius chose for a feasible solution: he measured the two angles between Pieterskerk-house-Hooglandse Kerk and Pieterskerk-house-Town Hall. Taking the angle Town Hall-house-Hooglandse Kerk instead of one of these two would have yielded a similar solution.

   In geometrical terms, the problem is now:

   Problem 3.1 (Resection Problem) *Given three points in position, P, T and H, $\angle POH = \alpha$ and $\angle POT = \beta$. It is required to determine OP, OT and OH (see figure 3.1).*

2. Snellius first gave the general idea of the solution. He argued that O lies on the intersection of two given circles, one drawn on the chord PH with circumference angle α and a second drawn on the chord PT with circumference angle β (cp. *Elements* III.33).

3. Next, Snellius gave an exact geometrical construction of the point O, by constructing these two circles. He remarked that there was no other solution than O, because two circles cut each other in at most two points (of which P also had to be one). If the problem had originated from pure geometry, Snellius would have been ready.

4. Now that it was a practical problem, two extra steps were necessary in order to find a solution in numbers. One by one, Snellius considered a number of triangles in the figure, of which three properties (angles or sides) were known, from which the other three could be deduced. He was still dealing with the general case, not with a numerical example. The sought distances were now 'given'.

5. Snellius reworked all the steps of his argument with the numerical values which he had measured and calculated previously.

   He could have considered to rework his calculations after replacing α or β by $\angle TOH$ and then to take the average value of his results to minimize the influence of measurement errors, but this line of thought seems not to have occurred to him.

Before and after Snellius, other mathematicians solved this or similar problems independently. The Greek astronomer Hipparchus solved a problem from astronomy that was mathematically equivalent to problem 3.1. It is found in Ptolemy's work and could have been known to Snellius, but he may not have thought of it while working on the resection problem. A later contribution was made by Laurent Pothenot, who published a solution in 1692. He was incorrectly assumed to have solved the problem for the first time and therefore the problem was often called after him in later times. One of the more recent approaches was to describe the problem in a Cartesian coordinate system, and to generalize it to more given points.[41]

In 1960, the Delft surveyors' society *Snellius* placed a memorial stone in the building currently occupying the space where Snellius's house formerly stood, to commemorate the first resection (the building, in the Doezastraat, is now a medical centre).[42] The present whereabouts of this stone are unknown, but a new plaquette has been fixed to the building by the same society in 2005.

Ad 9. Snellius now knew the distance between Alkmaar and Bergen op Zoom and their relative positions (latitude and difference in longitude). One of the other data needed to reach the desired result was the value of π. Snellius gave the approximations of Viète, Romanus and Van Ceulen, in many more digits than his measuring accuracy necessitated. Through a long series of calculations he arrived at the end of his quest: the length of one degree on the meridian of Alkmaar was 28,500 rods (a. 107.33 km), and therefore the length of a meridian 10,260,000 rods, about 38,639 km. This is about 3.65 % less than the modern value.[43]

[41] [Haasbroek, 1968, pp. 110–111], [van der Plaats, 1889, pp. 31–38], [Wolf, 1973a, pp. 181–182], [Tropfke, 1923, pp. 97–98], [Haerpfer, 1910, pp. 14–20].

[42] [Haasbroek, 1968, pp. 112–113].

[43] [Snellius, 1617b, pp. 198, 212–217], [Haasbroek, 1968, p. 108].

He published his work in the *Eratosthenes Batavus* of 1617. The full title is *Eratosthenes Batavus De Terrae ambitus vera quantitate, a Willebrordo Snellio, διὰ τῶν ἐξ ἀποστημάτων μετρουσῶν διοπτρῶν, suscitatus*, which means 'The Dutch Eratosthenes, on the true size of the circumference of the earth, recalled from the grave by means of optical instruments according to measured distances'. Snellius announced clearly in whose footsteps he followed by the explicit reference to his Greek forerunner.

3.4 After publication: improvements

Ad 10. Van Musschenbroek tells us that Snellius discovered some mistakes in his book after its publication when teaching his students practical geometry. Snellius showed them how he had taken measurements in the fields around Leiden to calculate the distances between the villages in the neighbourhood. After his discovery, he decided to start all over again and redo all the measurements and calculations, maintaining the same base line.[44]

The new results are found in the Brussels copy of the *Eratosthenes Batavus*, written in Snellius's own hand, and they were published by Van Musschenbroek in the first part of his treatise. Pages 185–192 of *Eratosthenes Batavus*, which contain an important part of the observations and calculations of the network, are lacking in the Brussels copy, and consequently we do not have a direct source of Snellius's corrections there. However, Van Musschenbroek had at his disposal these changes and included them in his book.[45] Not all Snellius's changes were improvements: some new errors were introduced.[46]

Ad 11. According to Van Musschenbroek, Snellius decided to publish a corrected and extended version of the *Eratosthenes Batavus* when he found out that the book had shortcomings. The completion of this second edition was thwarted by a unique chance to measure a more accurate base line when in January 1622, during a very rough winter, the fields and meadows around Leiden were inundated and it started to freeze. In this way, Snellius had a large, perfectly smooth surface at his disposal. He measured a new base line with a chain near Voorschoten on 3 February. He repeated his measurement three times for extra precision, then used it to determine anew the distance between two towers in Leiden and Zoeterwoude, which turned out to be almost 5 rods (a. 18.8 m) more than in his former calculations.[47]

[44][van Musschenbroek, 1729, p. 358], French translation of pp. 358–361 in [Bosmans, 1900, pp. 114–116].

[45]There are more indications that Van Musschenbroek had a further reworked copy of the *Eratosthenes Batavus*; cp. [Bosmans, 1900, pp. 118–119]. However, the description of the Belgian extension of the triangulation network lacks in his book.

[46][Haasbroek, 1968, p. 66].

[47][van Musschenbroek, 1729, pp. 358–359, 401], Haasbroek discusses three new base lines of

No details about this new base line can be traced in Snellius's preliminary work for the new edition, but he does write that a few years after his first measurements, he seized the opportunity to determine the distance between Leiden and Noordwijk when it was freezing, and that this distance would be his line of reference from then on, instead of that between Leiden and The Hague.[48] However, no calculations connecting this new line to his other observations are known.

After having explained these extensions by Snellius, Van Musschenbroek wrote that at this point Snellius would have had to calculate all the triangles for the third time; worn out by all the labour, Snellius had just remarked that the new base line (near Voorschoten) had to be preferred to that in *Eratosthenes Batavus* and 'he did nothing further'.[49] It seems odd that Snellius would have ended his enormous project before reaching the finish, especially because adjusting his triangle network to this new base line did not involve as much travelling as his last round of observations. If Snellius had indeed decided to stop at this point, method must have mattered more for him than result. It seems more likely, however, that he did not have the time to finish the job immediately, therefore postponed it and did not have the chance to come back to it before he died only a few years later, in 1626.

In the second part of his treatise, Van Musschenbroek used this new base line and Snellius's observations (some of them checked by him) to calculate an improved approximation of the circumference of the earth. Haasbroek thought that Van Musschenbroek falsified some of Snellius's observations to make the computations more consistent.[50] He concluded too sternly:

> With this falsification Van Musschenbroek's work is fully condemned; it is entirely unreliable and it contrasts very badly with the faithful work carried out by Snellius a century earlier.[51]

Ad 12. Snellius wanted to improve his results in yet another way, extending his net of triangles to Mechlin in the Southern Netherlands in order to improve his final result by using two places further apart than Alkmaar and Bergen op Zoom. He included Antwerpen, Hoogstraten and Mechlin in his network, and he used the polar altitude determined in Mechlin in 1615. The other necessary

1622: [Haasbroek, 1968, pp. 70, 79–87].

[48]'Nam initio distantiam inter Leidam et Noortwicum non eramus ex ipso fundamento dimensi; quod tamen aliquot annis post ad certiorem operis rationem oblata oportunitate, hiberno tempore per glaciem commodissime absolvimus.' [Snellius, 1617c, p. 183].

[49]'adeoque iterum ab initio calculus omnium Triangulorum erat repetendus: pertaesus procul dubio laboris, quo iam bis perfunctus erat Auctor, tantum adnotavit, ultimae modo mensurae fidendum esse, non primae, quam in Eratosthene statuit, atque ulterius fecit nihil [...]', [van Musschenbroek, 1729, p. 359].

[50][Haasbroek, 1968, pp. 68, 79].

[51][Haasbroek, 1968, pp. 83–84].

measurements were probably done in 1625. Van Musschenbroek thought that this addition had been lost, but it is included in the Brussels copy of *Eratosthenes Batavus*.[52] Apparently, there were no serious obstacles related to travelling in hostile territory, nor even to climbing towers and surveying the country. This extension did not lead to a new final conclusion either.

Ad 13. The correspondence between Snellius and Gassendi from 1625 shows that Snellius was still acquiring new material then, notably on the sizes of feet and ells in different places, because, as Snellius wrote, 'my Eratosthenes has to be augmented and when it has been enriched, it must be published'.[53]

3.5 Conclusion

Snellius's efforts to measure the earth were huge. He had to make many travels, followed by numerous calculations. The transport of his instruments, especially of the largest quadrant, must have been a tremendous job, as they were heavy. Yet Snellius took the large quadrant with him al the way to Mechlin, trusting this instrument more than whichever he could have borrowed there. This shows his determination in making his observations as exact as possible and his willingness to solve practical problems if an important matter was at stake.

Moreover, this whole measuring expedition probably cost Snellius some money. Part of the expenses may have been paid by the Sterrenbergs and Snellius's dedication to the States General yielded him 200 guilders.[54] This was a considerable amount, 40 % of his annual salary in 1617. Yet it is quite likely that his costs exceeded this amount and that he had to supply it from his own not too full pocket. The remuneration for Snellius was not financial, but intellectual and social.

Snellius showed both signs of pride of his achievements and awareness of his shortcomings. The latter induced him to work on an improved version of *Eratosthenes Batavus* continually, after wrapping up his research and writing the book rather quickly at first. He shared many of his considerations and activities

[52][van Musschenbroek, 1729, pp. 358, 361]; extension in [Snellius, 1617c, ad p. 208-1–7], edited in [Bosmans, 1900, pp. 121–126].

The chronology is not completely clear. Van Musschenbroek dates the Belgian extension between the publication of *Eratosthenes Batavus* and 1622. In the new chapter which Snellius wrote about this extension, he mentioned an astronomical observation done in Mechlin in 1615 and the death of Coignet, who died 'in the previous year 1624', [Snellius, 1617c, ad p. 208-6], without making explicit which events belong to which year. In fact, Coignet died 24 December 1623, [Quetelet, 1873]. All taken together, 1625 is the most likely year for the triangulation in the Southern Netherlands; if Snellius had done all this work in 1615, he certainly would have included it in the first edition of his book.

[53]'Eratosthenes meus augendus, et locupletior est in lucem edendus.' [Gassendi, 1964, p. 393].

[54][Smit, 1975, p. 241], [Dodt van Flensburg, 1848, p. 16].

with his readers, which is best exemplified by the digression on the tragic fate of Oudewater and his lengthy explanation of the experiment to determine the relative density of water. This expansiveness might be a figure of speech: Snellius may have meant to incite the reader to empathize with him and to appreciate his immense project more.

Eratosthenes Batavus is indeed an epitome of Snellius's *oeuvre*, not only because of its contribution to surveying, but also because of its programmatic value: Snellius showed in it how fruitful a combination of bookish scholarship and practical research could be.

3.6 Reception

Some examples may suffice to show the reception of the *Eratosthenes Batavus*. A practical consequence of Snellius's project was the frequent later use of the Snellius mile for navigational purposes. This mile was defined as one fifteenth of the length of one degree on the meridian, that is 22,800 Rhenish feet (a. 85.864 km). The *Eratosthenes Batavus* was not directly accessible to sailors because of the language barrier, but a portion of the results could be learned through vernacular texts. Snellius's measure of the circumference gained wide acceptance.[55] In this way Snellius had contributed to the solution of the pressing problem of the absence of standard measures, to which he had paid much attention in the book. Just as he had stated in his dedicatory letter, the work was useful for navigation, yet it did not contribute substantially towards solving the longitude problem.

Snellius introduced a new development of the technique of triangulation in his book, yet this did not reach the Dutch surveyors for a long time because they did not master Latin either. As they could not read the book, they may have considered its topic as too esoteric for their purposes, evidently unaware of the relevance of Snellius's work for making local maps. The distances between places that he had calculated were not used either.

Abroad, a number of scientists read Snellius's work and although they had some criticisms, they used the method for their own degree measurements or they selected other results. For instance Kepler mentioned Snellius's reduction of longitude differences in Western Europe in his *Tabulae Rudolphinae*, and he also referred to *Eratosthenes Batavus* in the *Somnium*. Wilhelm Schickard asked Matthias Bernegger in Strasbourg on behalf of Kepler and himself to send them his (Bernegger's) local foot and ell expressed on a solid, not moistened piece of paper, so that they could use Snellius's comparison of these measures with Roman ones. Between 1624 and 1635, Schickard also surveyed the duchy of

[55] [Davids, 1986, pp. 113–114, 126, 150, 212, 270–271, 310].

Württemberg using the principles of triangulation as developed by Snellius.[56]

Snellius's work also reached map makers abroad. For instance Tobias Mayer used Snellius's data in several maps of the Low Countries in the eighteenth century.[57]

[56] [Pouls, 1997, pp. 261–264], [Kepler (F. Hammer ed.), 1969, pp. 95, 98], [Kepler (V. Bialas and H. Grössing eds.), 1993, p. 362]; [Seck, 2002, 1, p. 172], [Betsch, 1996, pp. 136–142].

[57] [Mayer, 1747], [Mayer, 1748a], [Mayer, 1748b]. [Mayer, s a] contains some of Mayer's own notes on *Eratosthenes Batavus*.